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ON NEW SIMPSON TYPE INEQUALITIES FOR THE p-QUASI CONVEX FUNCTIONS

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Abstract. In this paper, we establish some new Simpson type inequalities for the class of functions whose derivatives in absolute values at certain powers are p-quasi-convex functions.

1. Introduction

A function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is said to be convex if the inequality

$$f\left(tx+(1-t)y\right) \leq tf\left(x\right)+\left(1-t\right)f\left(y\right)$$

is valid for all $x, y \in I$ and $t \in [0, 1]$. If this inequality reverses, then f is said to be concave on interval $I \neq \emptyset$. This definition is well known in the literature.

It is well known that theory of convex sets and convex functions play an important role in mathematics and the other pure and applied sciences. In recent years, the concept of convexity has been extended and generalized in various directions using novel and innovative techniques. For some inequalities, generalizations and applications concerning convexity see [1, 2, 4–6, 18, 21].

In [10], the author gave definition harmonically convex and concave functions as follow:

Definition 1.1. Let $I \subset \mathbb{R}\backslash\left\{0\right\}$ be a real interval. A function $f : I \to \mathbb{R}$ is said to be harmonically convex, if

$$f\left(\frac{xy}{tx+(1-t)y}\right) \leq tf\left(y\right)+(1-t)f(x)$$

for all $x, y \in I$ and $t \in [0, 1]$. If this inequality is reversed, then f is said to be harmonically concave. The following result of the Hermite-Hadamard type inequality holds for harmonically convex functions.

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Definition 1.2. A function $f : [a, b] \to \mathbb{R}$ is said quasi-convex on $[a, b]$ if

$$f\,(tx + (1-t)y) \leq sup\,\{f\,(x)\,, f(y)\}$$

for any $x, y \in [a, b]$ and $t \in [0, 1]$.

Definition 1.3. Let $I \subset (0, \infty)$ be a real interval and $p \in \mathbb{R} \backslash \{0\}$. A function $f : I \to \mathbb{R}$ is said to be a p-convex function, if

$$f\left([tx^p + (1-t)\,y^p]^{\frac{1}{p}}\right) \leq tf\,(x) + (1-t)\,f\,(y)$$

for all $x, y \in I$ and $t \in [0, 1]$. If this inequality is reversed, then f is said to be p-concave.

Hermite-Hadamard inequality for the p-convex function is following:

Theorem 1.1. *Let $f : I \subset (0, \infty) \to \mathbb{R}$ be a p-convex function, $p \in \mathbb{R} \backslash \{0\}$, and $a, b \in I$ with $a < b$. If $f \in L\,[a, b]$ then we have*

$$f\left(\left[\frac{a^p + b^p}{2}\right]^{\frac{1}{p}}\right) \leq \frac{p}{b^p - a^p}\int_a^b \frac{f\,(x)}{x^{1-p}}dx \leq \frac{f\,(a) + f\,(b)}{2}.$$

These inequalities are sharp [5, 8].

Definition 1.4. A function $f : I \subseteq \mathbb{R}_+ \to \mathbb{R}_0$ is said to be a harmonically quasi-convex function on I if

$$f\left(\left(\frac{\lambda}{x} + \frac{1-\lambda}{y}\right)^{-1}\right) \leq sup\,\{f\,(x)\,, f(y)\}$$

holds for all $x, y \in I$ and $\lambda \in [0, 1]$.

Definition 1.5. Let $I \subset (0, \infty)$ be a real interval and $p \in \mathbb{R} \backslash \{0\}$. A function $f : I \to \mathbb{R}$ is said to be p-quasi-convex, if

$$f\left([tx^p + (1-t)\,y^p]^{\frac{1}{p}}\right) \leq max\,\{f\,(x)\,, f\,(y)\}$$

for all $x, y \in I$ and $t \in [0, 1]$. If this inequality is reversed, then f is said to be p-quasi-concave [17].

It can be easily seen that for $p = 1$ and $p = -1$, p-quasi convexity reduces to ordinary quasi convexity and harmonically quasi convexity of functions defined on $I \subset (0, \infty)$, respectively. Moreover every p-convex function is a p-quasi-convex function.

Many papers have been written by a number of mathematicians concerning inequalities for different classes of harmonically convex, harmonically quasi-convex, p-convex and p-quasi-convex functions see for instance the recent papers [3, 7–12, 14, 17, 19, 20, 22–24] and the references within these papers.

The following integral inequality, named Simpson's integral inequality, is one of the best known results in the literature.

Theorem 1.2. *(Simpson's Integral Inequality). Let $f : I = [a, b] \subset \mathbb{R} \longrightarrow \mathbb{R}$ be a four time continuously differentiable on I°, where I° is the interior of I and $\left\|f^{(4)}\right\|_{\infty} < \infty$. Then*

$$\left|\frac{1}{3}\left[\frac{f\,(a) + f(b)}{2} + 2f\left(\frac{a+b}{2}\right)\right] - \frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{1}{2880}\left\|f^{(4)}\right\|_{\infty}(b-a)^4.$$

There are substantial literature on Simpson type integral inequalities. Here we mention the result of [11, 13, 16] and the corresponding references cited therein.
Throughout this paper we will use the following notations:

Let $0 < a < b$ and $p \in \mathbb{R}\backslash\{0\}$.

$$\begin{aligned}
A_p &= A_p(a,b) = \frac{a^p+b^p}{2}, \quad A_1 = A = A(a,b) = \frac{a+b}{2} \\
M_p &= M_p(a,b) = A_p^{\frac{1}{p}} = \left[\frac{a^p+b^p}{2}\right]^{\frac{1}{p}}, \\
I_t(x, A_p; u, v) &= \frac{\left|t-\frac{1}{3}\right|^u}{\left[(1-t)x^p + tA_p\right]^{v-\frac{v}{p}}}, \\
C_{x,M_p}(f) &= max\left\{\left|f'(x)\right|, \left|f'(M_p)\right|\right\}
\end{aligned}$$

where $t \in [0,1]$, $x \in [a,b]$ and $u, v \geq 0$.

2. Main Results

In this section, we will use the following Lemma to obtain our main the results [15]:

Lemma 2.1. *Let $f : I \subset (0,\infty) \longrightarrow \mathbb{R}$ be a differentiable mapping on I° (interior of I) and $a, b \in I^\circ$ with $a < b$ and $p \in \mathbb{R}\backslash\{0\}$. If $f' \in L[a,b]$, then the following equality holds:*

$$\begin{aligned}
&\frac{1}{6}\left[\, f(a) + 4f(M_p) + f(b)\right] - \frac{p}{b^p-a^p}\int_a^b \frac{f(x)}{x^{1-p}}dx \\
= &\frac{b^p-a^p}{4p}\left[\int_0^1 \frac{t-\frac{1}{3}}{\left[(1-t)a^p + tA_p\right]^{1-\frac{1}{p}}} f'\left(\left[(1-t)a^p + tA_p\right]^{\frac{1}{p}}\right)dt\right. \\
&\left. + \int_0^1 \frac{t-\frac{2}{3}}{\left[(1-t)A_p + tb^p\right]^{1-\frac{1}{p}}} f'\left(\left[(1-t)A_p + tb^p\right]^{\frac{1}{p}}\right)dt\right].
\end{aligned}$$

Theorem 2.1. *Let $f : I \subset (0,\infty) \longrightarrow \mathbb{R}$ be a differentiable mapping on I° (interior of I) and $a, b \in I^\circ$ with $a < b$ and $p \in \mathbb{R}\backslash\{0\}$. If $f' \in L[a,b]$ and $|f'|^q$ is p-quasi-convex on I for $q \geq 1$, then the following inequality holds:*

$$\left|\frac{1}{6}\left[f(a) + 4f(M_p) + f(b)\right] - \frac{p}{b^p-a^p}\int_a^b \frac{f(x)}{x^{1-p}}dx\right|$$

$$\leq \frac{b^p-a^p}{4p}\left[C_{a,M_p}(f)\,D_p(a,b) + C_{b,M_p}(f)\,E_p(a,b)\right]$$

where

$$D_p(a,b) = \int_0^1 I_t(a, A_p; 1, 1)\ dt, \qquad E_p(a,b) = \int_0^1 I_{1-t}(b, A_p; 1, 1)\ dt.$$

Proof. Using Lemma 2.1 and the Power mean inequality, we have

$$
\begin{aligned}
&\left|\frac{1}{6}\left[f(a)+4f(M_p)+f(b)\right]-\frac{p}{b^p-a^p}\int_a^b\frac{f(x)}{x^{1-p}}dx\right| \\
\leq\ & \frac{b^p-a^p}{4p}\left[\int_0^1\frac{\left|t-\frac{1}{3}\right|}{\left[(1-t)a^p+tA_p\right]^{1-\frac{1}{p}}}\left|f'\left(\left[(1-t)a^p+t\left(A_p^{\frac{1}{p}}\right)^p\right]^{\frac{1}{p}}\right)\right|dt\right] \\
&+\frac{b^p-a^p}{4p}\left[\int_0^1\frac{\left|t-\frac{2}{3}\right|}{\left[(1-t)A_p+tb^p\right]^{1-\frac{1}{p}}}\left|f'\left(\left[(1-t)\left(A_p^{\frac{1}{p}}\right)^p+tb^p\right]^{\frac{1}{p}}\right)\right|dt\right] \\
=\ & \frac{b^p-a^p}{4p}\left[\int_0^1 I_t(a,A_p;1,1)\ \left|f'\left(\left[(1-t)a^p+tM_p^p\right]^{\frac{1}{p}}\right)\right|dt\ \right] \\
&+\frac{b^p-a^p}{4p}\left[\int_0^1 I_{1-t}(b,A_p;1,1)\left|f'\left(\left[(1-t)M_p^p+tb^p\right]^{\frac{1}{p}}\right)\right|dt\right] \\
\leq\ & \frac{b^p-a^p}{4p}\left[\int_0^1 I_t(a,A_p;1,1)\quad dt\right]^{1-\frac{1}{q}} \\
&\times\left[\int_0^1 I_t(a,A_p;1,1)\left|f'\left(\left[(1-t)a^p+tM_p^p\right]^{\frac{1}{p}}\right)\right|^q dt\right]^{\frac{1}{q}} \\
&+\frac{b^p-a^p}{4p}\left[\int_0^1 I_{1-t}(b,A_p;1,1)\,dt\right]^{1-\frac{1}{q}} \\
&\times\left[\int_0^1 I_{1-t}(b,A_p;1,1)\left|f'\left(\left[(1-t)M_p^p+tb^p\right]^{\frac{1}{p}}\right)\right|^q dt\right]^{\frac{1}{q}} \\
\leq\ & \frac{b^p-a^p}{4p}\left[C_{a,M_p}(f)\,D_p(a,b)\ +C_{b,M_p}(f)\,E_p(a,b)\right].
\end{aligned}
$$

This completes the proof of theorem. □

Corollary 2.1. *Under conditions of Theorem 2.1,*

i. If we take $p=1$, then we obtain the following inequality for quasi-convex function:

$$
\begin{aligned}
&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]-\frac{1}{b-a}\int_a^b f(x)dx\right| \\
\leq\ & \frac{5}{36}(b-a)\,A\left(C_{a,M_1}(f),C_{b,M_1}(f)\right).
\end{aligned}
$$

ii. If we take $p=-1$, then we obtain the following inequality for harmonically quasi-convex function:

$$
\begin{aligned}
&\left|\frac{1}{6}\left[f(a)+4f\left(\frac{2ab}{a+b}\right)+f(b)\right]-\frac{ab}{b-a}\int_a^b\frac{f(x)}{x^2}dx\right| \\
\leq\ & \frac{b-a}{4ab}\left[C_{a,M_{-1}}(f)\,D_{-1}(a,b)\ +C_{b,M_{-1}}(f)\,E_{-1}(a,b)\right].
\end{aligned}
$$

Theorem 2.2. *Let $f:I\subset(0,\infty)\longrightarrow\mathbb{R}$ be a differentiable mapping on I° (interior of I) and $a,b\in I^{\circ}$ with $a<b$ and $p\in\mathbb{R}\backslash\{0\}$. If $f'\in L[a,b]$ and $|f'|^q$ is p-quasi-convex on I for*

$q>1$, $\frac{1}{r}+\frac{1}{q}=1$, *then*

$$\begin{aligned}&\left|\frac{1}{6}\left[f\left(a\right)+4f\left(M_p\right)+f\left(b\right)\right]-\frac{p}{b^p-a^p}\int_a^b\frac{f\left(x\right)}{x^{1-p}}dx\right|\\ \leq\ &\frac{b^p-a^p}{4p}\left[N_{p,r}^{\frac{1}{r}}(a,b)C_{a,M_p}\left(f\right)+O_{p,r}^{\frac{1}{r}}(a,b)C_{b,M_p}\left(f\right)\right]\end{aligned}$$

where

$$N_{p,r}\left(a,b\right)=\int_0^1 I_t\left(a,A_p;r,r\right)\ dt,\qquad O_{p,r}\left(a,b\right)=\int_0^1 I_{1-t}\left(b,A_p;r,r\right)\ dt.$$

Proof. From Lemma 2.1, Hölder's integral inequality and the p-quasi-convexity of $\left|f'\right|^q$ on $[a,b]$, we have,

$$\begin{aligned}&\left|\frac{1}{6}\left[f\left(a\right)+4f\left(M_p\right)+f\left(b\right)\right]-\frac{p}{b^p-a^p}\int_a^b\frac{f\left(x\right)}{x^{1-p}}dx\right|\\ \leq\ &\frac{b^p-a^p}{4p}\left(\int_0^1 I_t\left(a,A_p;r,r\right)dt\right)^{\frac{1}{r}}\left(\int_0^1\left|f'((1-t)\,a^p+tA_p)^{\frac{1}{p}}\right|^q dt\right)^{\frac{1}{q}}\\ &+\frac{b^p-a^p}{4p}\left(\int_0^1 I_t\left(b,A_p;r,r\right)dt\right)^{\frac{1}{r}}\left(\int_0^1\left|f'((1-t)\,A_p+tb^p)^{\frac{1}{p}}\right|^q dt\right)^{\frac{1}{q}}\\ \leq\ &\frac{b^p-a^p}{4p}\left[N_{p,r}^{\frac{1}{r}}(a,b)C_{a,M_p}\left(f\right)+O_{p,r}^{\frac{1}{r}}(a,b)C_{b,M_p}\left(f\right)\right].\end{aligned}$$

This completes the proof of theorem. □

Corollary 2.2. *Under conditions of Theorem 2.2,*

i. If we take $p=1$, then we obtain the following inequality for quasi-convex function:

$$\begin{aligned}&\left|\frac{1}{6}\left[f\left(a\right)+4f\left(\frac{a+b}{2}\right)+f\left(b\right)\right]-\frac{1}{b-a}\int_a^b f\left(x\right)dx\right|\\ \leq\ &\frac{b-a}{6}\left[\frac{1+2^{r+1}}{3\left(r+1\right)}\right]^{\frac{1}{r}}A\left(C_{a,M_1}\left(f\right),C_{b,M_1}\left(f\right)\right).\end{aligned}$$

ii. If we take $p=-1$, then we obtain the following inequality for harmonically quasi-convex function:

$$\begin{aligned}&\left|\frac{1}{6}\left[f\left(a\right)+4f\left(\frac{2ab}{a+b}\right)+f\left(b\right)\right]-\frac{ab}{b-a}\int_a^b\frac{f\left(x\right)}{x^2}dx\right|\\ \leq\ &\frac{b-a}{4ab}\left[N_{-1,r}^{\frac{1}{r}}\left(a,b\right)C_{a,M_{-1}}\left(f\right)+O_{-1}^{\frac{1}{r}}\left(a,b\right)C_{b,M_{-1,r}}\left(f\right)\right].\end{aligned}$$

Theorem 2.3. *Let $f:I\subset(0,\infty)\longrightarrow\mathbb{R}$ be a differentiable mapping on I° (interior of I) and $a,b\in I^{\circ}$ with $a<b$ and $p\in\mathbb{R}\setminus\{0\}$. If $f'\in L\left[a,b\right]$ and $\left|f'\right|^q$ is p-quasi-convex on I for*

$q>1,\ \frac{1}{r}+\frac{1}{q}=1,$ *then*

$$\begin{aligned}&\left|\frac{1}{6}\left[f\left(a\right)+4f\left(M_{p}\right)+f\left(b\right)\right]-\frac{p}{b^{p}-a^{p}}\int_{a}^{b}\frac{f\left(x\right)}{x^{1-p}}dx\right|\\ \leq\ &\frac{b^{p}-a^{p}}{12p}\left[\frac{1+2^{r+1}}{3(r+1)}\right]^{\frac{1}{r}}\left[C_{a,M_{p}}\left(f\right)Q_{p,q}^{\frac{1}{q}}\left(a,b\right)+C_{b,M_{p}}\left(f\right)S_{p,q}^{\frac{1}{q}}\left(a,b\right)\right]\end{aligned}$$

where

$$Q_{p,q}\left(a,b\right)=\int_{0}^{1}I_{t}\left(a,A_{p};0,q\right)dt,\qquad S_{p,q}\left(a,b\right)=\int_{0}^{1}I_{1-t}\left(b,A_{p};0,q\right)dt.$$

Proof. From Lemma 2.1, Hölder's integral inequality and the p-quasi-convexity of $|f'|^{q}$ on $[a,b]$, we obtain,

$$\begin{aligned}&\left|\frac{1}{6}\left[f\left(a\right)+4f\left(M_{p}\right)+f\left(b\right)\right]-\frac{p}{b^{p}-a^{p}}\int_{a}^{b}\frac{f\left(x\right)}{x^{1-p}}dx\right|\\ \leq\ &\frac{b^{p}-a^{p}}{4p}\int_{0}^{1}\left|t-\frac{1}{3}\right|\left|\frac{1}{\left[\left(1-t\right)a^{p}+tA_{p}\right]^{1-\frac{1}{p}}}f'\left(\left(1-t\right)a^{p}+t\left(A_{p}^{\frac{1}{p}}\right)^{p}\right)^{\frac{1}{p}}\right|dt\\ &+\frac{b^{p}-a^{p}}{4p}\int_{0}^{1}\left|t-\frac{2}{3}\right|\left|\frac{1}{\left[\left(1-t\right)A_{p}+tb^{p}\right]^{1-\frac{1}{p}}}f'\left(\left(1-t\right)\left(A_{p}^{\frac{1}{p}}\right)^{p}+tb^{p}\right)^{\frac{1}{p}}\right|dt\\ \leq\ &\frac{b^{p}-a^{p}}{4p}\left(\int_{0}^{1}\left|t-\frac{1}{3}\right|^{r}dt\right)^{\frac{1}{r}}\left(\int_{0}^{1}I_{t}\left(a,A_{p};0,q\right)\left|f'\left(\left(1-t\right)a^{p}+tM_{p}^{p}\right)^{\frac{1}{p}}\right|^{q}dt\right)^{\frac{1}{q}}\\ &+\frac{b^{p}-a^{p}}{4p}\left(\int_{0}^{1}\left|t-\frac{2}{3}\right|^{r}dt\right)^{\frac{1}{r}}\\ &\times\left(\int_{0}^{1}I_{1-t}\left(b,A_{p};0,q\right)\left|f'\left(\left(1-t\right)M_{p}^{p}+tb^{p}\right)^{\frac{1}{p}}\right|^{q}dt\right)^{\frac{1}{q}}\\ \leq\ &\frac{b^{p}-a^{p}}{12p}\left[\frac{1+2^{r+1}}{3(r+1)}\right]^{\frac{1}{r}}\left[C_{a,M_{p}}\left(f\right)Q_{p,q}^{\frac{1}{q}}\left(a,b\right)+C_{b,M_{p}}\left(f\right)S_{p,q}^{\frac{1}{q}}\left(a,b\right)\right].\end{aligned}$$

This completes the proof of theorem. □

Corollary 2.3. *Under conditions of Theorem 2.3,*

i. If we take $p=1$, then we obtain the following inequality for quasi-convex function:

$$\begin{aligned}&\left|\frac{1}{6}\left[f\left(a\right)+4f\left(\frac{a+b}{2}\right)+f\left(b\right)\right]-\frac{1}{b-a}\int_{a}^{b}f\left(x\right)dx\right|\\ \leq\ &\frac{b-a}{6}\left[\frac{1+2^{r+1}}{3\left(r+1\right)}\right]^{\frac{1}{r}}A\left(C_{a,M_{1}}\left(f\right),C_{b,M_{1}}\left(f\right)\right).\end{aligned}$$

ii. If we take $p=-1$, then we obtain the following inequality for harmonically quasi-convex function:

$$\left|\frac{1}{6}\left[f(a)+4f\left(\frac{2ab}{a+b}\right)+f(b)\right]-\frac{ab}{b-a}\int_a^b\frac{f(x)}{x^2}dx\right|$$
$$\leq \frac{b-a}{12ab}\left[\frac{1+2^{r+1}}{3(r+1)}\right]^{\frac{1}{r}}\left[C_{a,M_{-1}}(f)\,Q_{-1,q}^{\frac{1}{q}}(a,b)+C_{b,M_{-1}}(f)\,S_{-1,q}^{\frac{1}{q}}(a,b)\right].$$

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